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Determination of soil pressure on anti-landslide retaining structures based on the stress state analysis of a near-slope area

Détermination de la pression sur les structures contre glissements de terrain basée sur l'analyse d'etat du tension dans la région rampante

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ABSTRACT One of the preventative measures to reduce landslide hazard is the arrangement of retaining structures, for example, pile elements. To use pile parameters correctly it is necessary to know the value of the so-called landslide pressure. Most modern methods for the determination of the landslide pressure value are not always based on correct assumptions that allow us to reduce a statically indeterminate problem to a quasi-static one scores of times. An approach to the solution of the problem for the determination of the values of slope stability factor is set forth in this paper. The landslide pressure forces and active soil pressure forces acting on rigid enclosing structures are estimated on the basis of the analysis of the stress-strain state of the soil mass using the method of complex variable function theory and the FEM.

RÉSUMÉ L' une des mesures preventives de réduire les risques du glissement de terrain est un dispositif des configurations continentes, par exemple, des elements sur pilotis. Pour la determination correcte de dimensions des pals il faut connaître la valeur de la pression du glissement de terrain. La plupart des methodes modernes pour déterminer la pression du glissement de terrain est basée sur des hypothèses pas toujours correctes qui nous permettent de réduire plusieurs fois un problème statiquement indéterminé à un problème quasi-statique. Dans cet article, nous présentons une approche de résoudre le problème de la détermination de la valeur du coefficient d'équilibre du talus, les forces du glissement de terrain et des forces du pression du terrain actif sur les structures de protection rigides basée sur l'analyse de l'état du terrain de la déformation précontrainte par les méthodes de la théorie des fonctions d'une variable complexe et FEM.

1 STRESS DETERMINATION IN THE AREA UNDER STUDY

There is a well-known method (Muskhelishvili 1961) that enables the solution of the first basic elastic problem for the case when the elastic isotropic medium fills the simply connected region S, bounded by the simple contour L, in the plane $z=x+iy$ if there is the function $z=\omega(\zeta)$, fulfilling the conformal circle mapping $|\zeta|<1$ or the conformal half-plane mapping $\mathrm{Im}Z<0$ onto the area under study. The simplest solution is achieved when the function $z=\omega(\zeta)$ is polynomial (Ugodchikov 1966).

The following function (Bogomolov 1996) was proposed

$$z=\omega(\zeta)=C_0+C\zeta-\sum_{k=0}^{n}\frac{C_{2k+1}}{(\zeta+a-bi)^{2k+1}}, \qquad (1)$$

where, $z=x+iy$; $\zeta=\xi+i\eta$; C_0; C; C_1….. C_{2k+1} are any coefficients, including complex ones, a and b are real numbers, $b>0$. This function fulfills the conformal mapping of the lower half-plane $\mathrm{Im}Z<0$ onto the half-plane with a curvilinear boundary simulating the contours of slopes, backfalls, earth cuts earth fills of various configurations.

If the coefficients C_0; C; C_1….. C_{2k+1} are real numbers, the boundary of the simply connected area S is symmetric. If C_0; C; C_1….. C_{2k+1} are complex numbers, then it is asymmetrical.

The analytical solution of the first basic elastic problem for a weighty half-plane with a curvilinear boundary being under the action of external loads (Bogomolov 1996) was obtained by the mapping function (1) using the function theory method of complex variables (Muskhelishvili 1966). Furthermore, the coefficient of the soil lateral pressure ξ_0 can take on any value occurring in nature.

The functions entering into the known relations (Muskhelishvili 1966) and determining the numerical values of stress in the area under study were obtained in the following form (Bogomolov 1996)

$$\Phi(\zeta)=\frac{1}{\left[C+\sum\limits_{k=0}^{n}\frac{(2k+1)C_{2k+1}}{(\zeta+a-bi)^{2k+2}}\right]}\left\{J_1+\sum_{k=0}^{n}\frac{C_{2k+1}}{(2k)!}\times\rightarrow\right.$$
$$\left.\times\left[\sum_{s=0}^{m=2k+1}\frac{A_m^s\,\overline{\Phi(-a-bi)^{(m-s)}}}{(\zeta+a-bi)^{s+1}}-\sum_{s=0}^{m=2k}\frac{A_m^s\,\overline{\Phi'(-a-bi)^{(m-s)}}}{(\zeta+a-bi)^{s+1}}\right]\right\} \quad (2)$$

$$\Psi(\zeta)=\frac{1}{\left[C+\sum\limits_{k=0}^{n}\frac{(2k+1)C_{2k+1}}{(\zeta+a-bi)^{2k+2}}\right]}\left\{J_2+\left[\sum_{k=0}^{n}\frac{(2k+1)C_{2k+1}}{(\zeta+a-bi)^{2k+2}}\right]\times\rightarrow\right.$$
$$\times\Phi(\zeta)+\left[C_0+C\zeta-\sum_{k=0}^{n}\frac{C_{2k+1}}{(\zeta+a+bi)^{2k+1}}\right]\Phi'(\zeta)-\sum_{k=0}^{n}\frac{C_{2k+1}}{(2k)!}\times\rightarrow \quad (3)$$
$$\left.\times\left[\sum_{s=0}^{m=2k+1}\frac{A_m^s\,\Phi^{(m-s)}(-a-bi)}{(\zeta+a+bi)^{s+1}}-\sum_{s=0}^{m=2k}\frac{A_m^s\,\Phi^{(m+1-s)}(-a-bi)}{(\zeta+a+bi)^{s+1}}\right]\right\}$$

If the area under study is heterogeneous, or it is impossible to map it conformally, the methods of finite or boundary elements (Segerlind 1976, Ugodchikov & Khutoryanskii 1986) can be used for the analysis of its stress-strain state.

Thus, we believe that the stress fields σ_z, σ_x и τ_{zx} for the conditions of the problem under consideration are known.

2 DETERMINATION OF THE SLOPE STABILITY FACTOR

Let us write the Coulomb strength condition in the form proposed by Kako, introducing additionally some function K into it

$$K\tau_n=(\sigma_n+\sigma_\varepsilon)tg\varphi\,, \quad (4)$$

where, τ_n and σ_n are dimensionless (in γh fractions) tangential and normal stresses acting along some inclined site; K is some function of the stress state in the point of the soil mass, called a point stability factor, $\sigma_\varepsilon=C(\gamma h tg\varphi)^{-1}$ is the reduced cohesion pressure; C; φ; γ; and h are cohesion, the angle of internal friction, soil bulk density and the height of the slope, respectively.

When K=1, the expression (4) coincides with the More strength condition.

Let us express the stresses τ_n and σ_n in terms of their components σ_z; σ_x; τ_{xz}, the inclination angle of the site α and substitute the expressions obtained into the formula (4), then we get the following

$$K=\frac{\left[\frac{1}{2}(\sigma_z-\sigma_x)cos2\alpha+\frac{1}{2}(\sigma_x+\sigma_z)+\tau_{xz}sin2\alpha+\sigma_\varepsilon\right]tg\varphi}{\frac{1}{2}(\sigma_x-\sigma_z)sin2\alpha+\tau_{xz}cos2\alpha} \quad (5)$$

The shear inclination angle α, when K takes on the minimum value, is determined from the condition (6) by the formula (7) (Tsvetkov 1979):

$$\left.\begin{aligned}&\frac{\partial K}{\partial\alpha}=0;\\&\frac{\partial^2 K}{\partial\alpha^2}>0,\end{aligned}\right\} \quad (6)$$

$$sin2\alpha_{1;2}=-\frac{2\tau_{xz}}{B}\pm(\sigma_z-\sigma_x)\sqrt{\frac{B^2-D}{B^2D}} \quad (7)$$

where, $B=(\sigma_z+\sigma_x+2\sigma_{\text{св}})$; $D=4\tau^2{}_{xz}+(\sigma_z-\sigma_x)^2$.

The global stability factor of the slope along the most probable shear surface (MPSS) is determined by the following formula

$$K=\frac{\int_0^l F_{\mathrm{s,r}}(S)ds}{\int_0^l F_{\mathrm{a,r}}(S)ds} \quad (8)$$

where, $F_{s,r}$ and $F_{a,r}$ are confining and shearing forces in the points on the MPSS, determined by the numerator and denominator of the formula (5), respectively; S is the angular position of a point on the MPSS (Tsvetkov 1979).

It should be noted that the methodology used for the construction of the most probable sliding surfaces (Tsvetkov 1979; Bogomolov 1996) allows the immediate, without any additional calculations and constructions, discovery of a sliding surface with a minimum value of the safety factor, determined by the formulae (5-8).

3 DETERMINATION OF LANDSLIDE (ACTIVE) PRESSURE FORCES

If the stresses in the ground massif under study, for example, on a uniform slope, have been determined, and there are no plastic strain ranges, the determination of the landslide pressure is reduced to the following operational procedures:

a) Using the methodology (Tsvetkov 1979) and formulae (5-8), we plot the most probable damage line (MPDL) on the slope. It corresponds to the real physical and mechanical properties of the soil (C; φ; γ; ξ_o) and the geometrical dimensions of the area under study;

b) The position of the vertical section, which coincides, for example, with the axis of a pile restraining element, and in which it is assumed to determine the value of the landslide pressure forces, is specified.

c) Several points on the axis of the restraining element (including the point of intersection of this axis and the MPDL) are selected at equal distances from each other. Local ascending hypothetical damage lines (LAHDL) (Fig. 1) are plotted from these points by the same methodology. The local stability coefficients K_{st}^{l} and the corresponding sums of confining and shearing forces $F_{s,r}^{l}$ и $F_{a,r}^{l}$ are calculated.

d) The design value of the local stability coefficient K_{st} corresponding to that part of the MPDL, which is located above the target section (in Fig. 1 it is upward and to the right), is specified.

e) The "fictitious" confining forces $F_{s,r}^{f}$ and then the "fictitious" shearing forces $F_{a,r}^{f}$ for each local ascending hypothetical damage line are calculated. A "fictitious" confining force is equal in value to the sum of confining forces, provided that the stability coefficient along this line is equal to the design value, i.e. $K_{st}^{l} = K_{st}$ (this follows from the condition that the prism of destruction slides along the damage line as a single whole). The value of the "fictitious" shearing force is equal to the difference between the value of the "fictitious" confining force and the sum of confining forces acting along this line in reality.

f) Taking into consideration the fact that each curved slope is inclined to the selected axis at some angle α, we plot the component distribution diagrams of the horizontal and vertical forces $F_{a,r}^{f}$, acting on the axis of the restraining element. The horizontal component diagram of the "fictitious" shearing forces represents the landslide pressure diagram in the section under study. The vertical component diagram of the "fictitious" shearing forces is no other than the diagram of the "negative friction forces" that will affect the settlement of the pile restraining element.

We will explain the facts stated above by considering the concrete example.

Let a uniform curved slope have an angle of β=38^0 and the following characteristics of the soil: the angle of internal friction φ=12^0; the intercept cohesion C=0.039MPa; the soil density ρ=1810 kg/m^3; the lateral pressure coefficient for clay soil accepted equal to ξ_o=0.75 (Vyalov 1978). It is necessary to plot the force landslide pressure diagram in the cross-section A-A1 provided that the value of the design local stability coefficient is K_{st}=3.

The application software package ASV32 (Bogomolov et al. 1999), developed in Volgograd State University of Architecture and Civil Engineering, allows us to perform all of the procedures described above, and make all necessary calculations.

Figure 1 shows the design diagram of the method of complex variable function theory, in which the trace of the most probable shear surface, the traces of the local ascending hypothetical shear surfaces and the position of the axis A-A1 of the restraining element (RE) are seen.

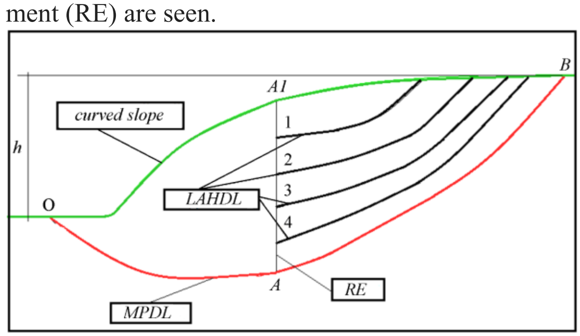

Figure 1.

It is believed that the prism of destruction slides along the destruction surface as a single whole, hence, the condition that all local stability coefficients are equal to the value of the design stability coefficient must be satisfied.

The "fictitious" confining and shearing forces are determined by the following formulae

$$F_{s,r}^{f} = \frac{K_{st}}{K_{st}^{l}} F_{s,r}^{l} \qquad (9)$$

$$F_{a,r}^{f} = F_{s,r}^{f} - F_{s,r}^{l} \qquad (10)$$

The results of calculations performed with the use of the program ASV32 are summarized in Table 1 for the convenience of analysis.

Table 1. Calculated values of confining and shearing forces.

LS	$F^f_{s,r}$ (γh)	$F^f_{a,r}$ (γh)	α^o	E_{ox} (γh)	E_{oz} (γh)
AB	20.6	6.97	22	6.64	2.15
4	17.9	5.3	22	5.16	1.65
3	9.26	0.83	20	0.78	0.28
2	5.96	-1.1	18	-1.1	-0.3
1	3.62	-3.6	8	-3.5	-0.5

If the "fictitious" shearing forces are negative, it means that they actually play the role of confining forces. Therefore, they are not taken into account when calculating the value of the landslide pressure.

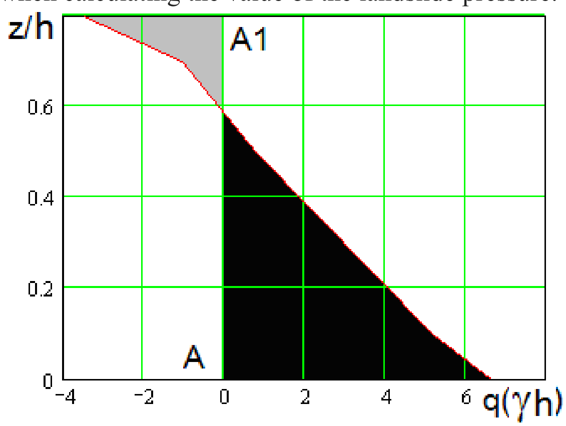

Figure 2.

Figure 2 shows the diagram of the landslide pressure for the example given above, which was plotted in the MahtCad shell according to the data presented in Table 1. The diagram is alternating, as follows from Figure 2, and consists of two curved triangles. The diagram of the landslide pressure is coloured in black, and it plays the role of the load when calculating the pile restraining element of the anti-landslide structure.

The "negative" part of the diagram, which is not included in the load when calculating the restraining element, is coloured in grey. If the diagram size and shape are known, it is easy to calculate the value of its resultant and determine the point of application of the latter. It should be noted that the vertical coordinates in Figure 2 have the dimension of h fractions of the height of the slope and the horizontal coordinates are measured in γh fractions.

4 DETERMINATION OF SOIL ACTIVE PRESSURE ON RIGID PIT ENVELOPE

To determine the value of soil pressure on rigid pit envelope we use the approach described above, which allows us to take into account the distributed surface load of any extent, intensity and position. The load intensity varies by an arbitrary law. The computational procedure is formalized in the computer program (Bogomolov et al. 2009), which has the state registration of the Russian Federation.

To create a database of an engineering method of assessment of lateral pressure forces on rigid pit envelope and retaining structures, the calculations and graphical constructions for homogeneous vertical slopes have been performed. The slope soil has the following characteristics: the angle of internal friction which changes in the range of φ=[10°-25°], the value of the given coherence pressure which changes in the ranges of σ_ε Є [0.4; 0.6; 0.8; 1.4; 2.0] depending on the slope height H, the value of s intercept cohesion C and the unit soil weight γ.

The coefficient of soil lateral pressure is taken equal to ξ_o=0.75, which corresponds to the average value for clay soils (Vyalov 1978).

The finite-element design scheme that consists of 14,100 finite elements conjugated in 7,227 nodes is shown in Figure 3. The stiffness matrix of the system is 136 in width.

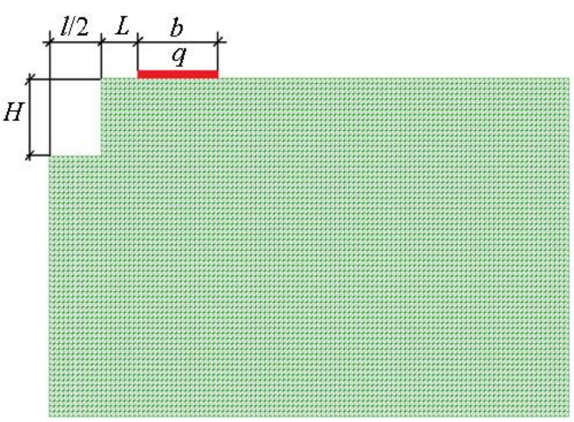

Figure 3.

The distribution of the external load along the day surface of the vertical slope is taken uniform. Its intensity varies in the range of q=0-3γH, and its width b and the position defined by the distance L from the edge of the slope varies within $b;L \in [0 - H]$.

The force diagrams of soil lateral pressure are resulted from the calculations for all possible combinations of numerical values of the design parameters. They are the following: the slope height is H=15m at q=0.75γH; b=H; L=H/2; γ=20 kN/m^3; φ=10^o; c=33кPa (σ_ε=0.637) and ξ_o=0.75γ, shown in Figure 4 as an example.

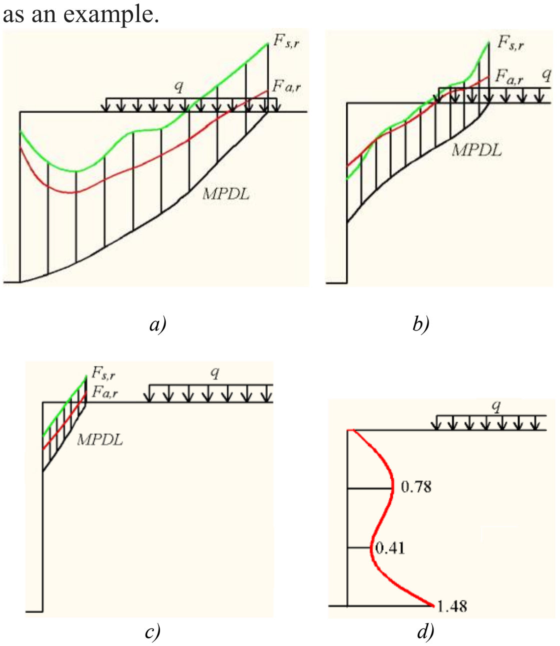

Figure 4.

Using a well-known technique of increasing the design values of specific cohesion and the tangent of the angle of internal friction by K/K_{st} times, let us determine such values of C and φ, under which the design value will take on the values of K=1.2; 1.1; 1.01. A hypothetically stable state of the slope in these cases will provide the "fictitious" confining forces that correspond to the soil strength properties increased by K/K_{st} times.

The elements of pit envelope are calculated for the values of "fictitious" shearing forces equal to the difference between the values of "fictitious" confining force and real confining force, calculated for the same slip line.

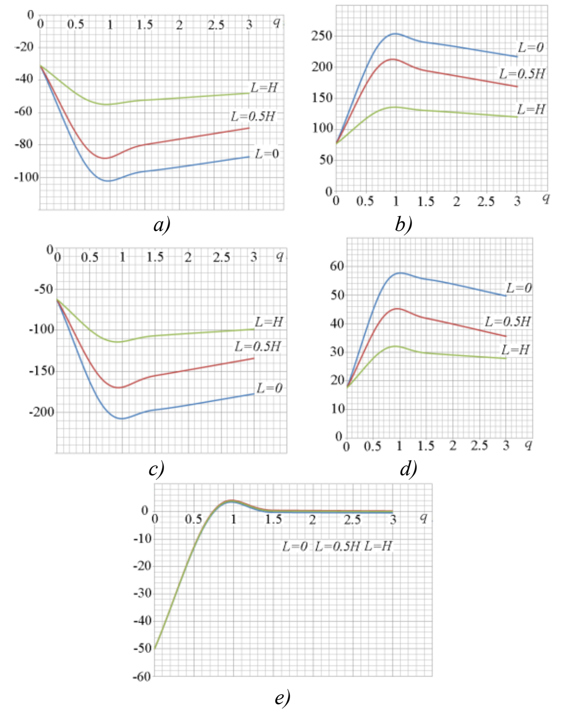

Figure 5.

It turned out that all the resulting diagrams of lateral pressure forces have a curved shape (Figure 4d), and the force generation curves can be approximated by the quintic polynomials with no less than 0.98 confidence:

$$q=(ay^5+by^4+cy^3+dy^2+ky)\gamma H \qquad (11)$$

where, a; b; c; d; k are dimensionless coefficients; y=Y/H; $y \in$ [0-1].

The results of the checking calculations showed that if the values of the coefficients k are considered not to depend on the value L and the arithmetical mean values of the coefficients when calculating are used, then the miscalculation will not exceed 2.23 %, and it will go to reserve.

Using interpolation methods, we can plot the diagrams of horizontal pressure on the rigid retaining structures for all possible variable combinations of design parameters that are described above.

Let us compare the results of calculation made by the proposed method with the experimental data obtained by other researchers.

There was proposed the method of experimental determination of soil pressure on retaining walls, which is reduced to the measurements of their deformation (deflection) (Rayuk 1961). Having defined the deflections in several points, we can analytically determine not only the pressure on the wall, but the bending and torque moments in any point in the wall by solving the inverse problem with the use of a variational (Kantorovich 1952) or energy (Wang Tszi-de 1959) method.

The experiment was conducted in the tray with dry sand of average size. The angle of internal friction of backfill sand was φ=36º, the volume weight was γ =17 kN/m^3. There were no data on the value of the coefficient of the backfill lateral pressure, so the value of ξ_o=0.4 was taken on (Bogomolov 1996).
A 6 mm thick plexiglass plate clamped on three sides and supported above was served as a model of a retaining wall. Its rigidity determined empirically was equal to D = 530 kgf·m.

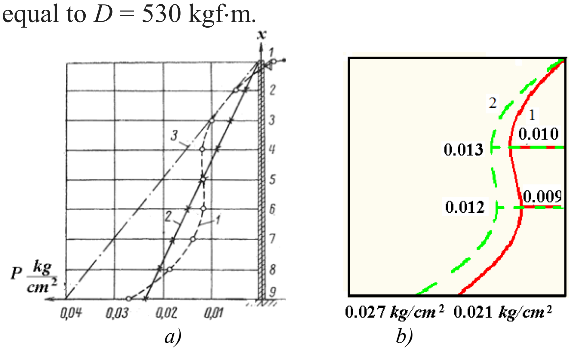

Figure 6.

Figure 6 shows the lateral pressure diagrams obtained experimentally (*a*) and by the proposed method (*b*).

The correlation of numerical values of ordinates of the appropriate diagram points shows that the values differ from each other by 20-22 %, approximately. At the same time, the shapes of the diagrams of the backfill lateral pressure, obtained experimentally (Rayuk 1961) (Figure 6*a*) and by the proposed method, are similar.

The resultants of the lateral pressure forces (they are numerically equal to the areas of the appropriate diagrams) are N_p=0.75 kg and N=0.57 kg. They differ from each other by 24 %, the value obtained experimentally being larger (Rayuk 1961).

As a result of additional calculations it was established that, if the value of the lateral pressure coefficient of the soil was taken equal to ξ_o=0.32, we would get the diagram of the lateral pressure that coincided totally with the diagram obtained by the experiment (Rayuk 1961).

5 CONCLUSIONS

The method for the force calculation of the active soil pressure on retaining and enclosing structures based on the stress-strain state analysis of the soil mass was proposed. The correlation of the experimental results for the determination of backfill soil pressure on the model of the enclosing structure and the value of the pressure, which was calculated on the basis of our proposals, showed that they were 22% different, but the shapes of the lateral pressure diagrams were absolutely similar.

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